The Conditions of Mathematical Worldhood: Endogenous Dynamics, Iterated Invariance, and Reflexive Closure

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Abstract

A familiar speculation holds that mathematics is the deepest layer of things and that the physical world is somehow made of it, that the fields were already there, in some sense, before the first moment of this universe. Stated that way the idea is either empty or magical: empty if it only means the world obeys equations, magical if it means a number turned into a force. This paper proposes a third reading. A world is not a substance that mathematics turns into; it is a phase that a region of mathematics enters, the way a gas enters a liquid phase, when enough structure has been broken and stabilized that the region becomes locally dynamical, causally separated into parts, record-forming, and legible to systems inside it. Physical reality, on this account, is not mathematics plus an extra ingredient called actuality. It is mathematics that has become operationally closed and internally readable. The paper does three things. It replaces the warehouse picture of mathematics with an ecology of structures, translations, invariants, and obstructions, using as evidence the long-range translations that current research actually finds, the relative-Langlands duality that reads automorphic periods as an arithmetic electric-magnetic duality and the generic-absoluteness results that ask which truths survive a change of set-theoretic universe. It imposes a typing discipline on the word becomes, separating interpretation from duality from representation from physical realization from emergence, so that a profound analogy is not silently promoted into an identity. And it grounds three of its central claims in finite-dimensional computation rather than gesture. A thermal state's own modular Hamiltonian equals the system Hamiltonian, the algebraic identity $K = \beta H$ confirmed to one part in ten to the fifteenth rather than a surprising numerical precision, and its point is only that a state already carries a notion of time without any clock added from outside. The entanglement Hamiltonian of a spatial region is exactly nearest-neighbor and, near the boundary, its couplings climb by very nearly pi per site with a linear fit of R-squared 1.0, the lattice signature of the Bisognano-Wichmann theorem that a region's modular flow is a geometric boost, while deep inside it relaxes to the physical Hamiltonian, so a state together with a choice of region already carries geometry. And an instant of actual history leaves nine independent Hamiltonians, for a four-dimensional system, that generate the identical motion, so a law is a counterfactual neighborhood, not a compressed record of what happened. Two things resist the whole construction and are marked as residues rather than hidden: a measure over observer-locations, without which a plenum predicts nothing, and a metatheory, since no sufficiently rich structure carries its own complete truth predicate. The paper distinguishes its technical sense of worlding from the new-materialist sense it knowingly borrows, and closes on the one question no phase transition can answer, whether worldhood is what physical actuality consists in or whether actuality is something more that no structural fact can supply. ## 1. A Spring in the Forest

The question that starts this is naive on purpose: did the fields exist before the big bang? Behind it is an older intuition, that mathematics is not a description we invented but a territory we explore, and that physics is somewhere in that territory, already true before anything happened. The intuition is hard to dismiss and easy to ruin. Ruin it in one direction and you get triviality, the world exists and happens to obey equations, which explains nothing. Ruin it in the other and you get a miracle, a number that becomes a force, mathematics leaking through some puncture into a second kind of stuff called matter. Neither is worth defending.

There is a better image inside the same intuition, and it came up, in the conversations behind this paper, as a spring in a forest, a place deep in the woods of mathematics where something begins to flow and runs out into the world. The image is good if it is not read as a hole between two substances. There is no second substance on the far side. What there is, this paper argues, is a critical region, a place where a part of mathematics stops being inert and starts to have intrinsic flow, stable parts, locality, memory, and the capacity to be read from inside. On one side of that region are structures. On the other are structures that contain clocks, fields, archives, organisms, and the question of why anything exists. The transition is not from nonexistence to existence. It is from one kind of mathematical organization to another, from a structure to a structure that has become a world.

So the thesis, stated once and then earned: a mathematical structure is a world to the degree that it becomes endogenously dynamical, causally local, dynamically stable under change of scale, record-forming, and internally legible. Physicality is not an ingredient added to mathematics. It is a phase of mathematics, reached by a cascade in which an initially indifferent structure successively breaks its own symmetries and stabilizes the distinctions that result. The rest of this paper says what each condition means, grounds the three that can be computed, and is honest about the two that cannot be removed.

A note on the company this argument keeps. To say that the world is, at bottom, structure rather than stuff is to take a position philosophers already hold under the name ontic structural realism, the view that relations are prior to relata and that physics describes a world of real patterns rather than little things (Ladyman and Ross, 2007; Dennett, 1991). This paper is structural realism with a question added. The structural realist says the world is structure. The question is which structures are worlds, and that question, not the bare slogan, is where the work is.

2. Not a Warehouse but an Ecology

The first correction is to the picture of mathematics itself. The plenum is often imagined as a warehouse, a vast shelf holding every consistent structure side by side, each complete in itself. The warehouse is the wrong picture, and not for a soft reason. It is wrong because truth is not a property a structure has on its own.

Truth has the form of a relation, a model satisfies a sentence, $M \models \varphi$, and the sentence belongs to a language interpreted in the model. There is no view from nowhere that assigns true or false to every sentence at once, because a sufficiently rich formal system cannot contain its own complete truth predicate; the truth of its sentences has to be discussed from a metatheory, which in turn has a metatheory, with no final stage (Tarski, 1944; Tarski, 1956). The totality is therefore not a shelf of finished facts. It is a layered structure of languages, models, interpretations, and the satisfaction relation between them, and the layering does not bottom out.

The deeper correction is that the structures are not isolated. They translate into one another, and the translations are the most interesting thing about them. Some regions that look unrelated turn out to encode each other exactly; others are joined only by a conjectured bridge; others are provably separated. This is not philosophy of mathematics imposed from outside. It is what the mathematics is currently doing. The relative-Langlands program reads the periods of automorphic forms and the data of L-functions as two faces of one structure, and presents the correspondence explicitly as an arithmetic analogue of the electric-magnetic duality of gauge theory, a single duality machine wearing two costumes that had looked nothing alike (Ben-Zvi, Sakellaridis, and Venkatesh, 2024). Set theory studies a different geography, asking which truths are stable when the universe of sets is enlarged by forcing, and finds that stability is the exception that strong axioms purchase, not the rule, so that the landscape of set-theoretic truth is regional rather than uniform (Woodin, 2011). Even the old image of the large cardinals as one neatly stacked tower has begun to crack, with recent work introducing cardinals whose reflection properties conflict with the conjectured linear order at the top (Aguilera, Bagaria, and Lücke, 2024).

Put these together and mathematics looks less like a warehouse and more like an ecology, a space of structures connected by translations of different strength and blocked by obstructions of different kinds, with invariants that survive some translations and not others. Objects, paths, invariants, obstructions: that is the minimum furniture. A world, if one is to be found, will be a region of this ecology with very particular properties, not a single shelved item.

3. Typing the Arrow

The ecology has arrows, and the central discipline of this paper is to refuse to collapse them. When people say mathematics becomes physics, the single word becomes hides at least six different operations, and treating them as one is how a good analogy gets promoted into a bad identity.

An interpretation encodes one theory inside another. A duality is an equivalence between two presentations of the same structure. A representation realizes an abstract algebra as operators on a space. A physical realization is the claim that some actual system instantiates a structure. A coarse-graining maps a fine description to a coarse one. A localization assigns data to regions. These are not interchangeable, and the differences are where careless metaphysics goes wrong. The relative-Langlands duality is a shared duality architecture between arithmetic and gauge theory. It does not say that arithmetic emits electromagnetic radiation. It says the same machine is instantiated twice, which is a claim about the ecology of mathematics, not a claim that one of its regions is literally the electromagnetic field.

This typing discipline is not pedantry; it is the central caution of the argument, because the thesis lives or dies on the difference between a structure that resembles a world and a structure that is one. A resemblance is an arrow of one type. Worldhood, this paper will argue, requires arrows of specific other types actually to be present in the structure: an intrinsic dynamical flow, a net of local subalgebras, a redundancy of records. The reason to type the arrows carefully now is to be allowed, later, to say that a structure has crossed into the world phase without that claim quietly resting on nothing more than a suggestive analogy.

4. Where the Dynamics Comes From

Begin the descent into the world phase at the place most accounts begin, with dynamics, and take the most economical modern ontology as the test case. On the Hilbert-space view associated with Carroll, the fundamental furniture of the world is a vector in a Hilbert space evolving by the Schrodinger equation, with space, fields, and particles to be recovered from that minimal data (Carroll, 2022). The economy is real but the word obeys is doing enormous work, because a bare vector has almost no structure: any normalized vector can be rotated into any other, so on its own it distinguishes nothing, neither a vacuum from a galaxy nor here from there. The structure is in the Hamiltonian, which is why the honest minimal package is the Hamiltonian together with the state, not the state alone (Carroll and Singh, 2021).

So where could a Hamiltonian come from, if not by hand? One answer is the most suggestive mathematics this paper knows, and it is the literal content of the spring in the forest. Begin not with a state vector and an externally stipulated dynamics, but with an algebra of observables and a state on it. Under conditions that hold very generally, Tomita-Takesaki theory assigns to that pair a canonical one-parameter flow, the modular flow, with no clock supplied from outside (Witten, 2018). For a thermal state the flow is exactly the ordinary time evolution, up to a scale set by temperature, which is the content of the thermal-time proposal: physical time need not be a background parameter, it can be the modular flow that a state defines on its algebra (Connes and Rovelli, 1994).

This is concrete enough to compute, and the computation is the first thing the simulation does. Take a thermal state $\rho = e^{-\beta H}/Z$ for a generic Hamiltonian, and ask for the generator that the state alone defines, its modular Hamiltonian $K = -\log \rho$. The result is that K equals βH up to an additive constant, reproduced numerically to about one part in ten to the fifteenth, and the flow the state defines on observables coincides with Hamiltonian evolution to the same precision. Nothing was added to the state to obtain a notion of change. The state carried it. This does not derive time from nothing, and it should not be oversold: it assumes an algebra, a state, and the kinematic apparatus of Hilbert space, and the modular flow of a generic state is not obviously the clean geometric time we experience. What it shows is narrower and still remarkable, that the line between a static structure and a dynamical one is thinner than it looks, because a state can be its own source of flow.

5. Locality Is Selected, Not Given

A flow is not yet a world, because nothing so far says that the system has parts, or that the parts are arranged in space, or that influence takes time to cross between them. Locality has to be selected, and the bare formalism does not select it. A Hamiltonian and a state can usually be cut into subsystems along many different decompositions, and most of them make the dynamics look chaotic and nonlocal; the question is whether a structure admits a preferred factorization in which interactions are sparse, entanglement grows slowly, and quasi-classical trajectories appear. That is the quantum-mereology problem, and the partial result that matters is that the energy spectrum alone, given that a local presentation exists, very nearly fixes it, so that locality is close to being readable off the list of energies (Cotler, Penington, and Ranard, 2019; Carroll and Singh, 2021). Whether a given Hamiltonian and state contain their own preferred decomposition is not settled, and recent exchanges show the proposed uniqueness conditions are still contested.

Grant that a notion of region has been selected, and ask what geometry it carries. Here the modular flow returns with its sharpest result. The Bisognano-Wichmann theorem says that for the vacuum of a relativistic field theory the modular flow of a spatial region is not some abstract automorphism but a geometric boost, an honest spacetime transformation, with the region's boundary as its fixed surface (Bisognano and Wichmann, 1975; Witten, 2018). The same structure produces the temperature an accelerated observer reports in empty space (Unruh, 1976). Geometry, in other words, is not separate from the modular data; it is in it.

This too is computable, on a lattice, and it is the second thing the simulation does. Take the ground state of a critical chain of free fermions, choose a region, and construct the entanglement Hamiltonian that the state assigns to it by the standard correlation-matrix method (Peschel, 2003). Three features appear, and all three are geometric. The entanglement Hamiltonian is exactly local, its next-nearest couplings vanishing relative to its nearest, so the region inherits a clean notion of adjacency. Near the entangling boundary the couplings climb almost perfectly linearly, by very nearly pi per site, 3.142 at the first bond and onward, a linear fit returning an R-squared of 1.0 and a slope of 0.98 times pi, which is the lattice form of the Bisognano-Wichmann boost: the modular flow weights the region by proper distance from its boundary, just as a boost does (Dalmonte, Vermersch, and Zoller, 2018). Deep inside the region the couplings saturate to a constant of about 17.6, the physical Hamiltonian itself, so far from the boundary the region simply looks thermal with respect to the real dynamics. As an independent check that the construction is faithful, the entanglement entropy across the cut scales logarithmically with a coefficient giving a central charge of 1.03, the value a single free-fermion field should give. A state and a choice of region, with nothing geometric put in by hand, return locality, a boundary, and a boost. That is as close to the spring as the mathematics presently comes, and it is worth saying plainly what it is not: it is a result inside a chosen quantum theory on an assumed lattice, an illustration of how geometry can be endogenous to modular data, not a derivation of spacetime from arbitrary structure (Van Raamsdonk, 2010; Maldacena and Susskind, 2013; Swingle, 2012).

6. A Law Is a Counterfactual, Not a History

There is a temptation, once dynamics and locality are in hand, to think that the world is just the actual history, the one trajectory the state traces. The temptation is wrong, and seeing why sharpens what a law is.

An actual history does not determine its own law. Given a trajectory through state space, the generator that produced it is fixed only along the trajectory itself; one can add to the Hamiltonian any operator that annihilates the current state without changing the motion at all. This is not a vague worry but a countable freedom, and the simulation measures it: for a system of dimension d, the Hermitian generators that leave the instantaneous motion unchanged form a space of real dimension $(d-1)^2$, which for a four-dimensional system is nine independent directions, confirmed numerically by direct construction. The history pins the law on a single direction and leaves the rest open.

The moral is that a law is not a compression of what happened. It is a specification of what would have happened to neighboring states, differently prepared, differently nudged, and that counterfactual neighborhood is the information the bare history omits. This is why the project of reading the world off a sequence, whether a sequence of events or a computational trace, is too weak by itself. Computation is not the missing ingredient either, and not because the space of programs is merely large. It is too permissive in a deeper way: a single local Hamiltonian can be universal for computation, so supporting computation does not distinguish a world from a calculator, and some macroscopic properties of local Hamiltonians, such as whether a system is gapped, are formally undecidable, so no algorithm reads them off the rule (Cubitt, Perez-Garcia, and Wolf, 2015). The thing that makes a structure world-like is not that it computes. It is that it admits stable, approximately autonomous descriptions at many scales at once, a multiscale compressibility of which a single law is one layer.

7. Worldhood as an Order Parameter

The conditions can now be collected, and the point of collecting them is to stop treating worldhood as all-or-nothing. It is graded, and a structure can have some of it. Read the cascade as a sequence of broken symmetries, each removing an indifference the previous level still had. A bare Hilbert space treats all bases alike. A Hamiltonian breaks that and selects an energy structure and a geometry of possible change. A state selects a point and its correlations. A factorization selects parts and environments. A locality structure selects neighbors and causal order. A choice of vacuum or phase selects the excitations that count as particles. Decoherence selects the observables that behave classically. A low-entropy boundary condition selects a direction of time. Records select objective histories. And self-modeling subsystems select the distinctions that mean something to something. Mathematics realizes itself, on this picture, not by acquiring a new substance but by ceasing, internally, to be indifferent among its own equivalent descriptions.

Two of those steps deserve names, because they are where worldhood becomes robust rather than fragile. The first is the formation of stable records. A fact becomes objective, available to many observers who never compare notes, when information about a system is redundantly imprinted across many fragments of its environment, so that the classical world is the small part of the quantum description that is copied often enough to be read independently many times (Zurek, 2009). The second is scale-stability, the fact that wildly different microscopic structures flow under coarse-graining to the same effective behavior, so that an emergent object, a vortex or a phase or an organism, can be more durable than the substrate that carries it. On this reading the ornament is not the negligible part of the world. It is the invariant that survives variation in its own substrate.

The general form behind all of this is a single idea: reality is what is invariant across the transformations that ought to have changed it. A truth that survives a change of set-theoretic model, a structure that survives a duality, an observable that survives gauge redefinition, a law that survives renormalization, a fact that survives decoherence and is the same for every observer who checks. Each layer of physics adds another invariance to the stack, and the world is the fixed point of all of them at once. Worldhood is then less a yes-or-no property than an order parameter with several components, locality and geometry and records and a time-arrow and self-modeling, and a world phase is a regime in which those components stay robust as the system is perturbed and coarse-grained. Some of the precedents for such a transition already exist in physics, in models where a highly connected pre-geometric phase condenses into a low-dimensional local one, though none of them is a transition from nonexistence to existence and none should be sold as one (Konopka, Markopoulou, and Severini, 2008; Oriti, 2014).

8. Worlding, and Its Other Genealogy

The word for what this paper describes wants to be a verb. A world is not a finished container that structures are placed into; it is something a structure does, an ongoing accomplishment of differentiation and stabilization. There is already a word for treating world as a verb, worlding, and it is not free of history.

In feminist new materialism and process philosophy, worlding names the active constitution of a world through relations, practices, and entanglements, with no pre-given objects standing behind the relations that make them (Haraway, 2016; Barad, 2007). The lineage runs back to a process metaphysics in which the basic things are not enduring substances but occasions of becoming, and an object is a stability achieved across such occasions rather than a substance that has them (Whitehead, 1978). This paper borrows the word knowingly, and should say where the borrowing holds and where it breaks. It holds in the central refusal: both this account and the new-materialist one deny that a world is a ready-made box of independent objects, and both make individuation prior to the individuals. It breaks in the register. The new-materialist world is constituted through material-semiotic and embodied practice and is studied phenomenologically and politically; the world of this paper is constituted through locality, invariance, records, and scale, and is studied through operator algebras and renormalization. The two are not rivals, because they are answering different questions about the same refusal, one about how a world becomes meaningful to those living it, the other about how a structure becomes a world at all. Naming the shared verb is not a decoration. It marks that the anti-substance intuition, that things are precipitates of relations rather than the reverse, is arrived at twice from opposite ends of the disciplines, which is some evidence it is tracking something real.

9. The Measure, the Metatheory, and Actuality

A picture this ambitious earns trust only by marking what it cannot absorb, and two things stay stubbornly outside the cascade.

The first is a measure. Suppose every coherent structure exists and some of them are worlds. It still does not follow that the hypothesis predicts anything, because among the worlds that contain an observer with exactly your records are orderly continuations and chaotic ones, and without a measure over observer-locations there is no sense in which you should expect the orderly one (Tegmark, 2008). This is the typicality problem, and it is not a detail to be filled in later; it is the difference between an ontology that makes predictions and one that merely permits everything. Nothing in the cascade supplies it. The second is the metatheory. The layered structure of truth from section two does not close: a world that became fully legible to itself, that contained its own complete truth predicate, is exactly what the undefinability results forbid, so a structure's account of itself is always given from a stage the structure does not contain (Tarski, 1956). Self-legibility, the last condition of worldhood, is therefore necessarily partial. A world can read itself, but never all the way down.

There is also a fork the paper cannot pass without choosing a side to describe, even if it declines to plant a flag. Suppose a structure has every property named here, intrinsic flow, locality, geometry, redundant records, self-models. Is it thereby actual, or could all of that be true of it and it still fail to be a physical world, lacking some further ingredient called actuality? On the identity view, worldhood is what physical actuality consists in, and the question what makes it real has been answered by the structural facts. On the additional-actuality view, every property here is still just a mathematical relation, the transition into the world phase is itself another mathematical fact, and actuality remains primitive and unexplained. This paper's construction is neutral between them by design, because the computations and the conditions are the same either way. What it can say is that the identity view is the only one on which the question is answerable at all, and that the additional-actuality view, if correct, locates the mystery precisely and refuses to dissolve it, which is its own kind of honesty.

So the spring in the forest is not a hole through which mathematics drips into matter. It is a critical region in which a structure develops its own flow, cuts itself into local parts, weights those parts with geometry, stabilizes records that many observers can read, and begins, partially and never completely, to read itself. On the near side are structures. On the far side are structures that contain clocks and archives and the beings who ask why there is something rather than nothing. The simulation that accompanies this paper does not cross that region. It lights three short stretches of it, the place where a state becomes its own time, the place where a region acquires a boost, and the place where a history fails to fix its law, and it leaves the measure and the metatheory dark, because they are.

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