Deriving Gravity's Strength: The Weakness of Gravity as a Single Unmatched Number

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Abstract

Newton's constant is measured rather than derived, and the temptation is to read its smallness as a brute fact, 38 orders of magnitude of accident. The temptation is wrong twice over. The decimal of G is unit-dependent and so cannot be the thing a theory derives; the quantity that survives a change of units is the gravitational coupling of the proton, $\alpha_G^{(p)} = Gm_p^2/(\hbar c)$, equal to about 5.9×10^{-39}, equivalently a Planck-to-proton mass ratio of about 1.3×10^{19}. Massless spin-2 consistency, through the soft-graviton theorem and the reconstruction of the Einstein interaction from self-coupling, fixes the form of gravity and the universality of its coupling, and stops there: the soft theorem is homogeneous, so any rescaling of the overall normalization is equally consistent. A limited proposition makes the gap exact. Over a one-parameter family of theories with a canonical Einstein-Hilbert term on a fixed flat-space matter action, every member shares identical non-gravitational physics while assigning a different value to $\alpha_G^{(p)}$, so no matter observable can determine it, and the missing information has to arrive from an ultraviolet completion, a global regularity condition, an anomaly, a topological quantization, or a unique vacuum. Quantum chromodynamics then answers a different and answerable question, why the proton is light rather than why gravity is weak: one-loop running turns a moderate coupling into an exponentially small confinement scale, with no number of order 10^{-19} anywhere in the input. A deterministic computation runs the strong coupling from its measured low-energy value up to the Planck scale, where α_s^{-1} reaches about 52, and down to its Landau pole at $\Lambda_{\text{QCD}} \approx 0.14$ GeV, and finds that transmutation carries the Planck scale down to the QCD scale across 19.94 base-10 orders while the proton sits a factor of about 6.6 above Λ_{QCD}. The same computation reproduces the illustrative unification-scale bookkeeping in which a proton-to-unification factor near 2.2×10^{-33} and a Planck-to-unification factor near 610 multiply back to $\alpha_G^{(p)}$; that split is scheme- and threshold-dependent, and is reported as bookkeeping. Collapsing every framework onto one generated scale leaves a single dimensionless unknown, the ratio $C_G/C_p \approx 2.6 \times 10^{18}$ that the common scale cancels out of, and induced gravity, asymptotic safety, extra dimensions, string vacua, holography, species counting, and anthropic selection each rename this one number while keeping a free modulus, trajectory coefficient, vacuum, counterterm, rank, or measure. No accepted realistic construction reaches a prediction of $\alpha_G^{(p)}$ calibrated on non-gravitational data alone. The weakness of gravity is one match, between the gravitational response and the matter-generated scale, still unmade.

One Number Behind the Hierarchy

Newton wrote a force law with a constant in front of it, and for two centuries the constant was a number to be measured. In field theory it stopped being a proportionality. The Einstein-Hilbert action holds G as the coefficient of the curvature scalar, $S_{\text{EH}} = (\bar{M}_{\text{Pl}}^2/2) \int \sqrt{-g}\, R$ with $\bar{M}_{\text{Pl}}$ the reduced Planck mass, and the field equations $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$ make G the coefficient relating stress-energy to spacetime curvature (Einstein, 1916). The factor 8π is a convention tied to the Newtonian limit. Asking whether G can be derived is therefore asking whether a microscopic theory can compute the coefficient of the curvature operator relative to the scales its matter sector generates.

The first move is to stop deriving the wrong thing. The SI value of G changes when the kilogram, metre, and second are redefined, so its decimal digits are an artifact of a unit system, and a dimensionful constant carries no theory-independent content of its own (Duff, Okun & Veneziano, 2002). What survives a change of units is a dimensionless ratio. Fix the test mass to the proton, stable and precisely known and tied to the confining scale of the strong interaction, and form

$$\alpha_G^{(p)} \equiv \frac{Gm_p^2}{\hbar c} = \left(\frac{m_p}{M_{\mathrm{Pl}}}\right)^2 = \frac{m_p^2}{8\pi\,\bar{M}_{\mathrm{Pl}}^2}.$$

The current CODATA adjustment gives $G = 6.67430 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$, the least precise of the familiar constants at a relative uncertainty of 2.2×10^{-5}, together with a proton mass known to parts in 10^{10} and exact values of c and $\hbar$ (Tiesinga et al., 2025). The three expressions above agree to machine precision and evaluate to $\alpha_G^{(p)} = 5.9 \times 10^{-39}$, equivalently $M_{\mathrm{Pl}}/m_p = 1.3 \times 10^{19}$ and $\bar{M}_{\mathrm{Pl}}/m_p = 2.6 \times 10^{18}$.

Now the smallness reads differently. The famous 38 orders belong to $1/\alpha_G^{(p)}$, and they are twice the 19.11 base-10 orders of a single mass ratio, M_{Pl}/m_p. One ratio, squared, produces the whole separation. There is one number to explain, and it is a ratio of two masses. Gravity between two protons is smaller than their electrostatic repulsion by a factor of order 10^{-36}, and the live question is why the proton mass sits so far below the Planck mass (Wilczek, 2001).

The history of the field warns against the obvious shortcut. Dirac noticed that the large dimensionless numbers of cosmology and the large ratio M_{Pl}/m_p are of comparable size, and proposed that G might vary with cosmic time to explain the coincidence (Dirac, 1937). The specific hypothesis is now tightly constrained, and its methodological lesson holds: a large dimensionless number demands dynamics or selection, and a formula that reproduces 10^{-39} by a fortunate choice of integers or epochs explains nothing unless the formula and its inputs were fixed before the comparison was made. The target is invariant, singular, and a ratio of two masses. The known physics reaches remarkably far toward it and stops at one exact object, which the sections below locate.

What Consistency Already Fixes

A great deal about gravity is not free. If a massless spin-2 particle exists and couples to matter, the emission of a soft graviton factorizes off any scattering amplitude, and the gauge redundancy of the polarization tensor combined with momentum conservation forces the couplings to every species to be equal (Weinberg, 1964). This is the equality of inertial and gravitational mass, derived rather than assumed, and it is a strong result: universality is a theorem about massless spin-2 rather than an empirical coincidence appended to a force law. A free spin-2 field must then couple to the stress-energy it carries itself, and iterating that requirement, under locality, Lorentz invariance, and two-derivative dynamics, rebuilds the nonlinear Einstein interaction from the linear one (Deser, 1970). Below the Planck scale the whole structure is a predictive quantum effective field theory, with the Einstein-Hilbert term as its leading operator and calculable long-distance quantum corrections (Donoghue, 1994).

A single quantity is left untouched by all of it. The soft-graviton condition is homogeneous in the coupling: if a value κ is admissible, so is $\lambda\kappa$ for any λ, because rescaling every graviton vertex rescales both sides of the consistency relation together. Universality fixes that all species couple alike; it says nothing about how strongly. In the effective theory the same fact appears as the status of the Einstein-Hilbert coefficient, a renormalized Wilson coefficient whose value is measured or matched to a deeper theory and left unpredicted by the low-energy expansion. Consistency determines the operator R, its nonlinear completion, and the equality of coupling across matter. The number multiplying R stays open.

The Infrared Cannot Fix the Normalization

The gap can be stated as a theorem, provided the theorem is kept modest. It forbids nothing about quantum gravity in general. It states only that one particular kind of information, infrared matter physics together with spin-2 consistency, is insufficient by itself, and it points at where other information must enter.

Consider the family of theories built from a fixed matter Lagrangian by adjoining an Einstein-Hilbert term of variable strength,

$$S_M[g,\psi] \;=\; \int d^4x\,\sqrt{-g}\,\left[\tfrac{1}{2}M^2 R \;+\; \mathcal{L}_m(g,\psi)\right] \;+\; S_{\text{higher}},$$

and linearize about flat spacetime with a canonically normalized graviton,

$$g_{\mu\nu} \;=\; \eta_{\mu\nu} + \frac{2}{M}\,h_{\mu\nu}, \qquad \mathcal{L}_{\text{int}} \;=\; -\frac{1}{M}\,h_{\mu\nu}T^{\mu\nu} + O(M^{-2}).$$

Proposition 1 (Infrared normalization underdetermination)**.** *Let the matter sector be a fixed consistent non-gravitational quantum field theory, let gravity below a cutoff be a local diffeomorphism-invariant effective theory in which an Einstein-Hilbert operator is allowed, and impose no ultraviolet relation tying the matter and gravitational Wilson coefficients. Then the non-gravitational observables of the theory leave the coefficient of the Einstein-Hilbert term undetermined.*

Proof. Every member of the family S_M has the same flat-background matter action $S_m[\eta,\psi]$, so every gravity-excluded correlator, every mass, coupling, and cross-section computed without an external graviton, is identical across the family. The graviton enters matter amplitudes only through the interaction $-h_{\mu\nu}T^{\mu\nu}/M$, whose strength is set by M. Varying M rescales every gravitational amplitude while holding all non-gravitational data fixed. A single-valued map from those data to $\alpha_G^{(p)}$ would have to return different values at the same argument, and no such function exists. $\square$

The proposition is informative because it names its own escape routes. It fails, and $\alpha_G^{(p)}$ can be fixed, exactly when one of its premises is dropped: an ultraviolet completion that relates the two coefficients, a global regularity condition on the gravitational field space, an anomaly, a topological quantization of the coupling, or a uniquely selected vacuum. Every serious proposal to derive gravity's strength is, read through this lens, a proposal about which premise to break. The rest of this paper follows that thread.

Why the Proton Is Light

The one number is a mass ratio, and one half of it is understood. Non-Abelian gauge theories are asymptotically free: their coupling weakens at high energy and strengthens toward low energy (Gross & Wilczek, 1973; Politzer, 1973). A coupling specified at a high scale therefore runs, at one loop, as

$$\frac{\Lambda_{\mathrm{QCD}}}{\mu} \;=\; \exp\!\left(-\frac{2\pi}{b_0\,\alpha_s(\mu)}\right), \qquad b_0 = 11 - \tfrac{2}{3}n_f,$$

so that a moderate coupling at μ produces an exponentially smaller invariant scale Λ_{QCD} below it. This is dimensional transmutation, the trade of a dimensionless coupling for a dynamically generated mass (Coleman & Weinberg, 1973). The exponential is what does the work: nowhere in $\alpha_s(\mu)$, b_0, or μ does a number of order 10^{-19} appear, and yet the ratio it produces spans nearly 20 orders of magnitude. No fine-tuning is required. Most of the proton mass is then field energy of the confined gluon and quark system rather than the small current masses of the up and down quarks, so the proton stays heavy even in the limit of massless quarks, and its mass is $m_p = C_{\mathrm{had}}\,\Lambda_{\mathrm{QCD}}$ with C_{had} a pure number of order a few.

The reframing is due to Wilczek: take the Planck mass as primary and ask why the proton is light rather than why gravity is weak (Wilczek, 2001). Posed that way, the question has a real answer, conditional on the ultraviolet coupling, the field content, and the flavour thresholds. Asymptotic freedom explains the exponential smallness of m_p/M_{Pl} once those conditionals are supplied. It does not supply them, and it says nothing about the Planck scale it measures against. The achievement is a sharp localization of what remains unexplained, on the matter side a single ultraviolet coupling and a threshold ledger, and across the divide the normalization of gravity relative to that trajectory.

How Much of the Hierarchy Transmutation Buys

To see the localization in numbers, the accompanying computation integrates the one-loop renormalization-group equation for α_s with active-flavour thresholds, anchored on the measured value $\alpha_s(M_Z) = 0.1179$. Run upward through the top threshold to the Planck scale, and the coupling reaches $\alpha_s^{-1}(M_{\mathrm{Pl}}) = 52.48$, a moderate number near $1/50$. Run downward, matching b_0 across the bottom and charm thresholds, and the coupling diverges at a Landau pole $\Lambda_{\mathrm{QCD}} = 0.14$ GeV, the invariant scale of this illustrative one-loop scheme. The Planck scale stands 19.94 base-10 orders above that pole. The proton sits a factor $C_{\mathrm{had}} \approx 6.6$ higher still, so the net Planck-to-proton separation is the 19.11 orders the mass ratio records. Transmutation buys almost the entire hierarchy; the leftover hadronic factor is a number of order a few, well under a single order of magnitude, and no small input was smuggled in to produce the rest.

It is instructive to write the same $\alpha_G^{(p)}$ against a high matching scale, as long as the illustrative status of the exercise is kept in view. Choose a gauge-unification scale $M_X = 2 \times 10^{16}$ GeV. Then $(m_p/M_X)^2 = 2.2 \times 10^{-33}$ and $M_{\rm Pl}/M_X = 610$, and the product $(m_p/M_X)^2(M_X/M_{\rm Pl})^2$ reconstructs $\alpha_G^{(p)}$ to machine precision, assigning 32.66 of its 38.23 orders to the proton-to-unification hierarchy and the remaining 5.57 to the Planck-to-unification ratio. The decomposition is illuminating and not invariant. The scale M_X depends on the assumed particle content, the threshold spectrum, and whether exact unification occurs at all, and its factors move when those assumptions move. The same one-loop running that gives $\alpha_s^{-1}(M_{\rm Pl})$ near 52 gives $\alpha_s^{-1}(M_X) = 45.34$ in the Standard Model, a value that cannot be identified with the unified coupling near 25 of a supersymmetric spectrum without a full threshold ledger, because the two schemes run with different beta functions. Figure 1 draws both halves of the bookkeeping.

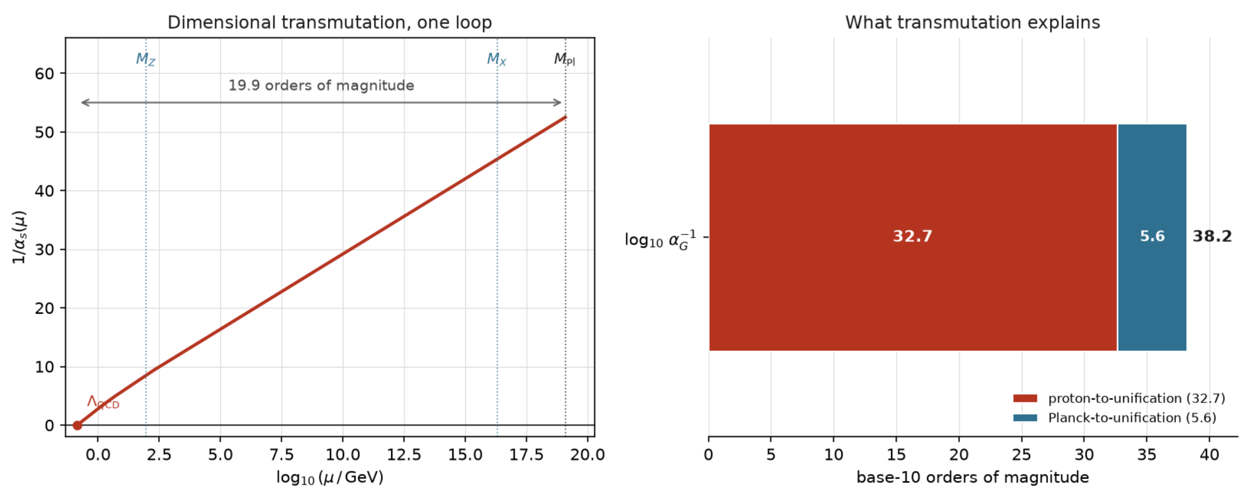

Figure 1: The order-of-magnitude ledger. Left: the one-loop running of $1/\alpha_s$ from the QCD scale, where it vanishes at the Landau pole, up to the Planck scale, where it reaches about 52; a coupling of a few hundredths at high energy sits 19.94 base-10 orders above $\Lambda_{\rm QCD}$, which is dimensional transmutation drawn to scale. Right: the 38.23 orders of $1/\alpha_G^{(p)}$ split into the 32.66 orders of the proton-to-unification hierarchy that QCD accounts for and the 5.57 orders of Planck-to-unification matching that it leaves open, for the illustrative choice $M_X = 2 \times 10^{16}$ GeV.

The numbers reproduce, the reconstruction closes, and none of it is a derivation of G. The reproduction is bookkeeping. The upward run takes a measured low-energy coupling as input; the downward pole is a scheme-dependent one-loop artifact standing in for the true confinement scale; and the whole ledger presumes a Planck scale on its high end, which is the quantity at issue. What the exercise establishes is the size and location of the residue. Almost 20 orders of the hierarchy are exponential and understood. The remainder is a matching condition between the gravitational scale and the top of the matter trajectory, a few orders of magnitude wearing, in this scheme, the costume of $M_{\rm Pl}/M_X$.

The Datum That Does Not Cancel

The unification-scale picture is convention-laden, so strip the convention away. Suppose a microscopic theory contains no fundamental mass and generates a single renormalization-group-invariant scale Λ_*. Every mass it produces is then Λ_* times a dimensionless coefficient, and in particular

$$\bar{M}_{\rm Pl} = C_G\,\Lambda_*, \qquad m_p = C_p\,\Lambda_*, \qquad \alpha_G^{(p)} = \frac{C_p^2}{8\pi\,C_G^2}.$$

The generated scale cancels in the ratio. Whatever Λ_* is, in GeV or in any other unit, it drops out, and the observable coupling depends only on the dimensionless quotient C_G/C_p. That quotient is the whole unsolved problem, stated without reference to any particular grand-unified model or matching scale. Its value is $C_G/C_p = \bar{M}_{\rm Pl}/m_p = 2.6\times10^{18}$, or inverted, $C_p/C_G = 3.85\times10^{-19}$, and the accompanying computation confirms that feeding this one ratio through $C_p^2/(8\pi C_G^2)$ returns the measured $\alpha_G^{(p)}$ with the scale entirely absent.

This is the object the infrared proposition said the matter sector could not reach, now written in its most portable form. It is a pure number, the relative normalization of the gravitational response coefficient against the coefficient of the confining matter scale, and it stays fixed under every choice of unit because the scale it might have held has cancelled. A theory that computed C_G and C_p in the same framework, from data that were themselves fixed, would have derived gravity's strength. Nothing short of that would.

Every Route Relaunders One Number

The proposed derivations are many, and read against the one datum they converge to a single shape: each supplies a mechanism that reorganizes the hierarchy, and each terminates in a renamed version of C_G/C_p. Scalar-tensor gravity promotes the Planck coefficient to a field value, $\bar{M}_{\rm Pl}^2 = F(\phi_0)$, converting the question of a constant into the question of a vacuum expectation value, its potential, and its cosmological endpoint (Brans & Dicke, 1961); precision equivalence-principle tests, which found no violation at the 10^{-15} level, sharply restrict the light composition-dependent scalars such constructions tend to introduce (Touboul et al., 2022). Classically scale-invariant and four-derivative gravity go further and generate the Planck scale by transmutation from dimensionless couplings, a genuine dynamical origin that still leaves the quartics, the non-minimal couplings, and a spin-2 ghost to be resolved (Salvio & Strumia, 2014). Induced gravity reads the Einstein term as a response coefficient of integrated-out matter, in the sense that elasticity emerges from molecular physics (Adler, 1982; Visser, 2002), and inherits the difficulty that the bare curvature operator is an independent allowed counterterm, so a matter loop computes a contribution to the coefficient rather than its total.

Asymptotic safety asks the coupling to flow to a nontrivial ultraviolet fixed point with finitely many relevant directions (Weinberg, 1979), a program made calculable by the functional renormalization group (Reuter, 1998), and it meets the datum as a relevant direction controlling the crossover to classical gravity, an integration constant equivalent to the Planck scale unless a global regularity condition discretizes it. Extra dimensions geometrize the ratio as $\bar{M}_{\mathrm{Pl}}^2 = V_n M_D^{n+2}$ and relocate the unknown into the stabilized compactification volume (Arkani-Hamed, Dimopoulos & Dvali, 1998). Holography is the one setting where a dimensionless bulk gravitational coupling is demonstrably encoded in boundary data, with a central charge proportional to L^{d-1}/G and large rank giving weak gravity, a genuine proof that Newton coupling need not be primitive (Maldacena, 1998), whose residue is the curvature-to-confinement ratio and the origin of the discrete rank. The pattern does not break. Emergent-graviton programs must first evade the theorem forbidding composite massless spin-2 particles in theories with a covariant conserved stress tensor (Weinberg & Witten, 1980), and then still compute the spin-2 residue against a baryon mass. Species arguments bound the gravitational cutoff by $M_{\mathrm{Pl}}/\sqrt{N}$ and inherit the origin of N (Dvali, 2010). Anthropic selection observes that stars and chemistry require gravity within a window (Carr & Rees, 1979), and a window is permission rather than prediction until an ensemble, a measure, and a conditioning are supplied. The recurrence across such different mathematics is the evidence. Each framework must add a global relation tying a gravitational response coefficient to a matter-generated scale, and local equations with local fixed-point data generically leave at least one continuous integration constant or vacuum choice behind.

What Would Count as a Derivation

It helps to grade the claims rather than argue them. Rewriting G in Planck units derives nothing. Deriving universality with a free coupling is structural, and it is real, and the coupling remains free. Expressing M_{Pl} through a vacuum value, a volume, a cutoff, or a central charge is a conditional scale relation, and generating a scale by transmutation with an unfixed trajectory is dynamical, and both are progress that stops short of a number. The level no accepted realistic construction has reached is the next one: fit dimensionless parameters to non-gravitational data alone, then predict $\alpha_G^{(p)}$ with no gravitational observable used anywhere in the calibration. Above it sits a strict derivation, in which uniquely fixed microscopic data yield $\alpha_G^{(p)}$ together with realistic confining matter and infrared general relativity. The controlled examples that come closest, the holographic ones, deliver a dimensionless bulk coupling in supersymmetric anti-de Sitter universes unlike our own, and stop short of the proton-to-Planck ratio of a world with confinement and chiral fermions.

The discipline that keeps the grading honest is a single rule applied recursively. A change of variables is not a derivation, so every proposed relation must be audited by asking what fixes the replacement quantity, and the audit continues until only derived quantities, independently measured non-gravitational inputs, or openly declared assumptions remain. Read $\bar{M}_{\mathrm{Pl}}^2 = \xi v^2$ and the question becomes ξ, the potential, the vacuum, and its radiative stability. Read $\bar{M}_{\mathrm{Pl}}^2 \sim N\Lambda_{\mathrm{UV}}^2$ and it becomes N, the physical cutoff, and the bare counterterm. The replacements are informative because they relocate the unknown to a sharper address, and misleading when the relocation is mistaken for a solution.

What the whole account leaves standing is a well-posed target rather than a mystery. Massless spin-2 consistency gives the form of gravity and the equality of its coupling. Asymptotic freedom gives an exponentially light proton. The observed $\alpha_G^{(p)}$ waits on one further relation, a global selection that turns the continuous family of gravitational normalizations into a unique or discrete value while producing realistic confining matter in the same theory. Until such a mechanism fixes C_G and C_p together, the smallness of gravity remains what this account has reduced it to: not a debt of 38 accidents to be paid one order at a time, but a single dimensionless match, between the gravitational response and the scale the matter makes for itself, that no theory has yet been made to strike.

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